

A mathematical model to develop a Scheffler-type solar concentrator coupled with a Stirling engine

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HIGHLIGHTS

- Mathematical model to develop a Scheffler-type solar concentrator.
- Equation to estimate the intercept factor.
- Comparisons between the parabolic dish and a Scheffler-type solar concentrator.
- Proposed Scheffler-type solar concentrator with advantages over parabolic dish concentrator.

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ABSTRACT

This study develops and applies a new mathematical model for estimating the intercept factor of a Scheffler-type solar concentrator (STSC) based on the geometric and optical behaviour of the concentrator in Cartesian coordinates, and the incorporation of a thermal model of the receptor is performed using numerical examinations to determine the technical feasibility of attaching the STSC to a 3 kWe Stirling engine. A numerical validation of the mathematical model is determined based on the experimental results reported for the WGA500 concentrator and the CNRS-PROMES system receiver. The numerical results allow for the design of the STSC and a comparison with a parabolic dish that provides the same thermal demand. Our findings show that the highest concentration was obtained with an edge angle of 45°, which was observed in the parabolic dish as well, but the STSC receiver shows a 7% increase in the thermal efficiency compared with the efficiency of the parabolic dish receiver. Finally, the STSC is appropriate for regions where the solar height allows for a reduction of convective thermal loss.

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1. Introduction

When generating electricity using alternative energy sources, it is appealing to incorporate technologies that concentrate solar thermal energy, such as heliostats, parabolic troughs and parabolic dishes connected to a Stirling engine [1]. The latest solar thermal technologies have high costs of installation, operation and maintenance and a decreased solar energy-to-electricity conversion efficiency [2]. Having a Stirling engine at the focal point of a parabolic dish decreases the effects of this efficiency problem by incorporating a solar concentrator with a fixed focal point. Scheffler developed a solar concentrator for solar cookers that are fixed inside the house [3]. This concentrator also has other applications, such as oil extraction [4], distillation [5] and sterilisation applications [6]. To incorporate this technology to generate electricity using Stirling engines, it is necessary to make some adjustments,

such as incorporating an azimuth axis that tracks the solar height and form continuously and incorporates the reflector surface with the least amount of imperfections, which is motivated by a study of ray tracing [7].

Different mathematical models have been developed using various mathematical tools and software to optimise the design of assembly systems. The model proposed by Harris and Lenz [8] analysed the thermal behaviour of a parabolic dish concentrator that used the view factor to estimate the amount of radiation reaching the receiver cavity in cylindrical, conical, elliptical and spherical receivers. Additional results for various concentrator geometrical shapes have been provided in Badescu [9]. Shuai et al. [10] used the Monte Carlo method to determine the performance of the radiation concentrated by a parabolic dish receiver for various geometries. Kumar and Reddy [11] presented a software implementation of a CFD (computational fluid dynamics) study to determine the optimal size of the aperture opening area of a spherical cavity coupled to a parabolic dish, which depends on the item's minimum convection. Chin-Hsiang and Hang-Suin [12] presented a study to

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Nomenclature

A	area (m ²)
a	normal distribution segment (–)
b	constant approximation to normal distribution (–)
C_{geo}	geometric concentration (–)
d	diameter (m)
f	focal length (m)
Gr	Grashof number (–)
h	convective heat transfer coefficient (W/m ² K)
I	direct solar irradiance (W/m ²)
K	fluid Thermal Conductivity (W/(m K))
L	thickness length (m)
p	distance from concentrator surface to focal point (m)
Pr	Prandtl number (–)
n	number of segment reflectors (–)
Nu	Nusselt number (–)
Q	energy flux density (W/m ²)
r	radius (m)
Ra	Rayleigh number (–)
Re	Reynolds number (–)
Sp	spacing (m)
S	separation (m)
T	temperature (K)
t	normal distribution variable (–)
w	width of the focal image (m)

Greek symbols

α_{eff}	effective absorbance of the cavity
ε	subtended angle of the sun
ε^*	emissivity

σ	standard deviation (mrad)
σ^*	Stefan–Boltzmann constant
η	efficiency (–)
ρ	surface reflectance (–)
φ	intercept factor (–)
θ	inclination angle of the cavity (°)
v	wind speed (m/s)
ψ	rim angle (°)

Subscripts

abs	absorber
amb	ambient
ap	aperture
ins	insulator
$cond$	conduction
$conv$	convection
cav	cavity
for	forced
Eff	effective
ext	outside
int	inside
rad	radiation
ref	reflector
rec	receiver
nat	natural
tub	pipe

optimise geometrical parameters for Stirling engines based on a theoretical analysis. Nepveu et al. [13] presented a model for a parabolic dish concentrator, known as a Eurodish, coupled to a 10 kW Stirling engine.

Halit et al. [14] presented an experimental study on the development of a beta-type Stirling engine for low and moderate temperature heat. Fraser [15] presented a model to estimate the performance of an Alpha-type Stirling engine, and the energy that is transmitted to the receiver is calculated by the equation proposed by Duffie and Beckman [16].

This equation states that the energy concentrated in the receiver of a solar concentrator is directly proportional to the direct radiation, the aperture opening area of the reflector, reflectivity and intercept factor. The latter concept is particularly useful in establishing the dimensions of the concentrator because reducing the receiver aperture opening diameter as a percentage of the solar image and incorporating the thermal model of the receptor defines the dimensions of the receiver for minimum heat loss.

There are different models available to estimate the intercept factor; Jaffe [17] and Badescu [18] presented a model that involves optical, thermal and conversion aspects of energy to optimise the dimensions of the parabolic dish concentrator. Romero [19] developed software for Sandia Laboratories that determined the energy that is intercepted by the circular or rectangular segments of a parabolic dish, which involves the optical aspects of the concentrator. Fraser [15] and Badescu [20] provided the results that are the most appropriate for parabolic dish concentrators that are coupled to a Stirling engine for different capacities. The models proposed by Badescu [21], Stine and Harrigan [22] considered both the influence of varying the receiver aperture opening diameter, which depends on the rim angle and height of the focal point, as well as the standard deviation of the errors caused by the geometrical factors and the optical concentration system.

After reviewing the literature, it was determined that no other previous model can be applied directly to evaluate the performance of an STSC because these models do not incorporate the geometric model of the STSC; therefore, in this study, a new mathematical model for an STSC coupled to a cavity receiver of a Stirling engine is presented. An analysis of the performance is carried out to compare an STSC and a parabolic dish to determine the technical viability of using this technology.

1.1. Description of an STSC

An STSC is the result of the interception of the open circular area with a small side section of a parabola, and this directs the solar radiation to a fixed focal point where a Stirling engine is installed and secured by a fixed support structure. It used a 2-axis tracking system, which consists of a daily tracking mechanism that moves the reflector mounted on a carriage in proportion to solar and other time-tracking mechanisms, which provides for rotation of the reflector that is synchronised with the movement of the sun during the day. This angular movement of the reflector is made around an axis oriented to maintain a fixed focal point normal to the incidence of the aperture opening area of the reflector, thus concentrating the solar radiation in the cavity that receives and heats a gas (helium, hydrogen or air) at high temperatures [23]. Later, the Stirling engine converts the heat into electricity, as observed in Fig. 1.

2. Materials and methods

2.1. Mathematical model

In this section, the mathematical model for attaching an STSC to a Stirling engine is developed using the mathematical estimation of

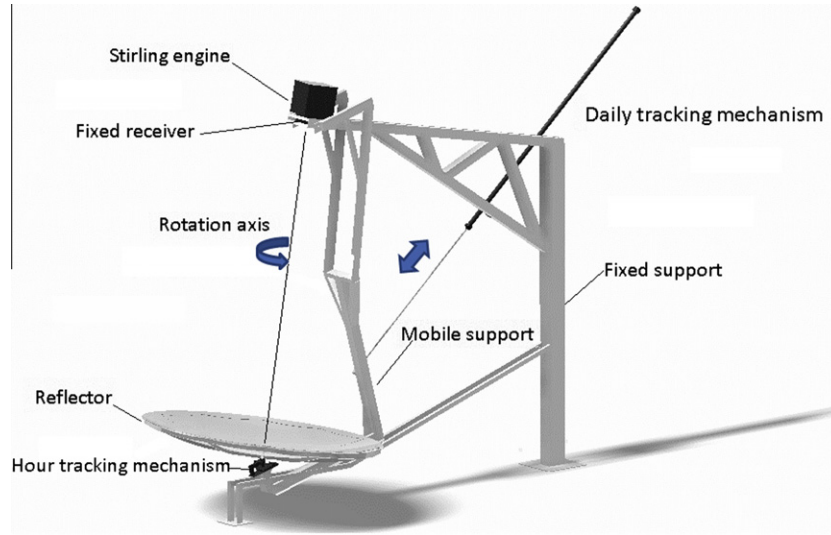


Fig. 1. STSC schematic.

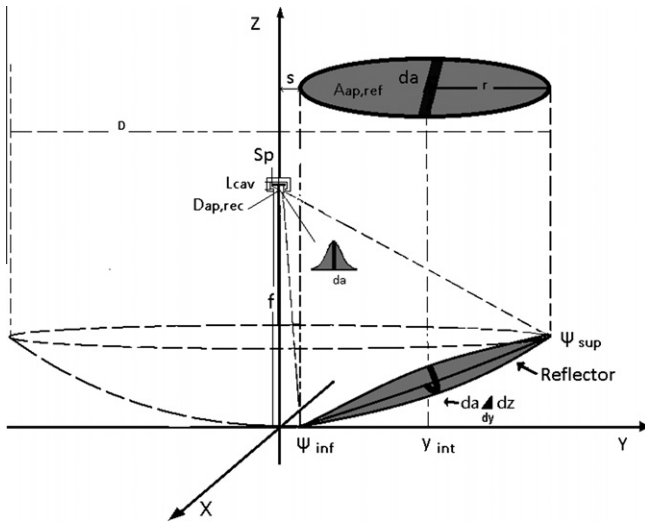


Fig. 2. Geometric scheme of an STSC.

the intercept factor for an STSC, which considers the optical and geometric models and incorporates the thermal model of a cavity receiver in accordance with the following considerations: the distribution of the solar image at the focal point corresponds to a normal distribution; the temperature inside the receiving cavity is evenly distributed; the heat transfer analysis is performed under stable conditions and is one-dimensional; the material properties remain constant; and the mathematical model begins by using Eq. (1), described by Duffie and Beckman [16], which estimates the amount of energy captured by a cavity receiver.

$$Q_{rec} = I_d A_{ap,ref} \rho \varphi \quad (1)$$

In Eq. (1), all of the terms are known, except for the intercept factor of the STSC, which is necessary to consider the geometric model of the STSC. Eq. (2) corresponds to the estimation of the intercept factor, and the total energy incident on the reflector (denominator) is easily established by substituting the value of the direct radiation and the aperture opening area. However, to determine the amount of energy intercepted (numerator), it is necessary to develop the

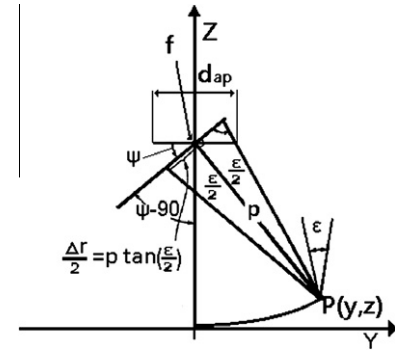


Fig. 3. Scheme of the solar image in the focal plane.

terms of the numerator, which involve the geometric and optical models of the STSC.

$$\varphi = \frac{\int_{-n}^{+n} I(a) da}{\int_{-\alpha}^{\alpha} I(a) da} \quad (2)$$

2.1.1. Geometric model

To develop the STSC model, we propose the integration of the geometric model, which corresponds to the intercept of a circular area with a virtual parabola, as shown in Fig. 2. In this geometrical model, it is necessary to specify the height of the virtual parabola, the coordinates of the centre of the circular area and the radius. This model moves the intercept point along the y-axis, which can be applied to parabolic concentrators, shifting the intercept point to the origin. The limits of the reflector are given on the y-axis and are delimited by $(y_{int} \pm r)$. Finally, the black-shaded section represents the differential area increase or the reflector segment, which corresponds to the area under the normal distribution curve of the solar image (i.e., the focal point).

The STSC geometric model is obtained by applying the surface integral for the interception of the two solids [24] and the total energy intercept, as observed in Eq. (3).

$$\int_{-n}^{+n} I(a) da = \int_n \int_{(R)} I(A) \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dx dy \quad (3)$$

Table 1

Comparison between the mathematical model and the experimental results.

Date of test	Mathematical model		Experimental CNRS	
	Value (kW)	Variation (%)	Value (kW)	Measurement
Intercepted energy	37.75	3.1	37.75	Reference value
Radiation losses	1.4	±25	1.71	In the range
Lost radiation emitted	2.59	±16	2.39	In the range
Convection losses	2.13	±50	1.98	In the range
Heat entering the motor	31.63	±4.3	31.67	In the range

To develop Eq. (3), it is necessary to define the equation of the parabola in Cartesian space, as observed in Eq. (4).

$$z = \frac{x^2}{4f} + \frac{y^2}{4f} \quad (4)$$

It is also necessary to define the equation of a cylinder in Cartesian space; Eq. (5) is used to delimit the area intercepted.

$$(x - 0)^2 + (y - y_{\text{int}})^2 = r^2 \quad (5)$$

By substituting Eq. (4) and Eq. (5) into Eq. (3), we obtain the differential area that can be assessed within the limits of the intercept of the parabola, and using the definition summation to replace the integral, we obtain the geometric model integration of the STSC, Eq. (6), which can be solved using an iterative method.

$$\int_{-n(y)}^{+n(y)} I(a) da = \sum_{y_{\text{int}}-r}^{y_{\text{int}}+r} I(a) \sqrt{k_1 y + k_2} \times \sqrt{r^2 y + 2y_{\text{int}} y - y_{\text{int}}^2 - r^2 y_{\text{int}} - y^2} \quad (6)$$

where $k_2 = \frac{4h^2 - y_{\text{int}}^2 - r^2 y_{\text{int}}}{4f^2}$ and $k_1 = \frac{(2y_{\text{int}} + r^2)}{4f^2}$

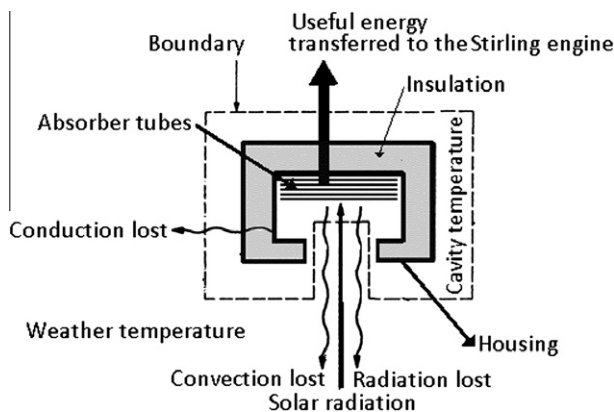
2.1.2. Optical model

To integrate the optical model [22] to Cartesian coordinates, as observed in Fig. 3, the variable $I(a)$ is replaced by the solar image in the function for the coordinates of the y -axis.

Eq. (7) involves the trigonometric and optical aspects of the solar image at the focal point and considers the statistical aspects that are due to the characteristics of the concentrator system components.

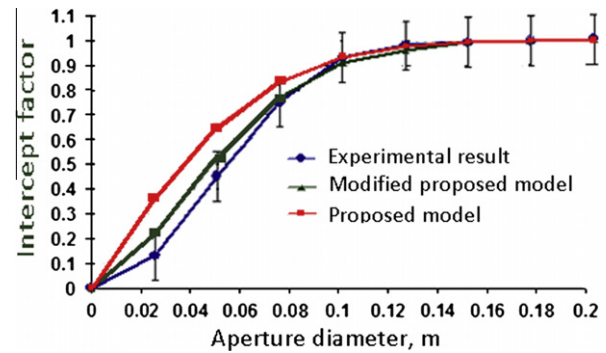
$$n(y) = \frac{2}{\sigma_{\text{tot}}} \tan^{-1} \left(\frac{d_{\text{ap}} \cos \psi}{2p} \right) \quad (7)$$

where $p = \sqrt{y^2 + (f - z)^2}$, is the normal distance from the reflector to the focal point, $\psi = \tan^{-1} \left(\frac{y}{f - z} \right)$, is the rim angle of the reflector

**Fig. 4.** Heat transfer in the cavity of the receptor.**Table 2**

STSC design parameters.

Design parameters		Operating conditions	
Number of absorber tubes	32	Direct solar radiation	900 W/m ²
Outside diameter of the tubes	0.63 cm	Air velocity	2.3 m/s
Material absorber tubes	SS 316	Ambient temperature	315 K
Absorber area	$6.44 \times 10^{-2} \text{ m}^2$	Solar maximum altitude	80.7°
Reflectance of the reflector	0.92	Fluid temperature	929 K
Insulation conductivity	0.042 W/(m K)	Heat transfer to fluid	8423 W
Insulation thickness	0.075 m	Total error	12 mrad

**Fig. 5.** Comparison of the proposed model, the modified proposed model and the experimental results.

$z = \frac{y^2}{4f}$, is the z -axis coordinate for the y -axis, and $\sigma_{\text{tot}} = \sqrt{(2\sigma_{\text{structure}})^2 + \sigma_{\text{sensor}}^2 + \sigma_{\text{drive}}^2 + \sigma_{\text{alignment}}^2 + (2\sigma_{\text{refl}})^2 + \sigma_{\text{sun}}^2}$, is the standard deviation of all the errors in the terms of the characteristics of the concentrator.

The normal distribution of the radiation at the focal point Eq. (8) is given by Abramowitz and Stegun [25]; however, there are more exact descriptions of the error distribution [21]. It is not necessary to include additional parameters that are not included in the present study. When considering the STSC, the maximum error of the approximation of the polynomial $I(a)$ is a continuous function $z(x)$, unlike the model proposed by Stine and Harrigan [22].

$$I(a) = 1 - \frac{2}{\sqrt{2\pi}} e^{-\frac{n(y)^2}{2}} (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + 2\text{res} \quad (8)$$

The intercept factor for the STSC is obtained by substituting the results of geometric, statistical and optical models, which depend on the variables of the dimensions and characteristics of the solar concentrator, as observed in Eq. (9).

$$\varphi = \frac{\sum_{y_{int}-r}^{y_{int}+r} I_d \left(1 - \frac{2}{\sqrt{2\pi}} e^{-\frac{n(y)^2}{8}} (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + 2res \right) \left(\sqrt{k_1 y + k_2} \sqrt{r^2 y + 2y_{int} y - y_{int}^2 - r^2 y_{int} - y^2} \right)}{I_d(\pi r^2)} \quad (9)$$

Allowing Eq. (9) to represent the geometry of a parabolic dish concentrator located at the origin ($y_{int} = 0$), the intercept factor for a parabolic dish concentrator in Cartesian coordinates is given by Eq. (10).

$$\varphi = \frac{\sum_0^{y_{int}+r} I_d \left(1 - \frac{2}{\sqrt{2\pi}} e^{-\frac{n(y)^2}{8}} (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + 2res \right) \left(\sqrt{k_1 y + k_2} \sqrt{r^2 y + 2y_{int} y - y_{int}^2 - r^2 y_{int} - y^2} \right)}{I_d(\pi r^2)} \quad (10)$$

2.1.3. Receiver thermal model

The receiver model was recently proposed by Fraser [15] and is obtained from the thermal analysis of a circular open cavity receiver with a flat absorber and thermal insulation in the walls. In this model, the receiver takes the energy from the reflector, and after

being affected by heat loss, the fluid is transferred to a Stirling engine that works through a cylindrical tube absorber plane, as shown in Fig. 4. This model assumes that the solar energy distribu-

tion is uniform in the cavity, that the optical and thermal properties of the receptor remain constant and that the thermal losses that occur in the cavity are due to the temperature difference between the cavity and the environment. The radiation and convection losses are due to the receiver aperture area and conduction

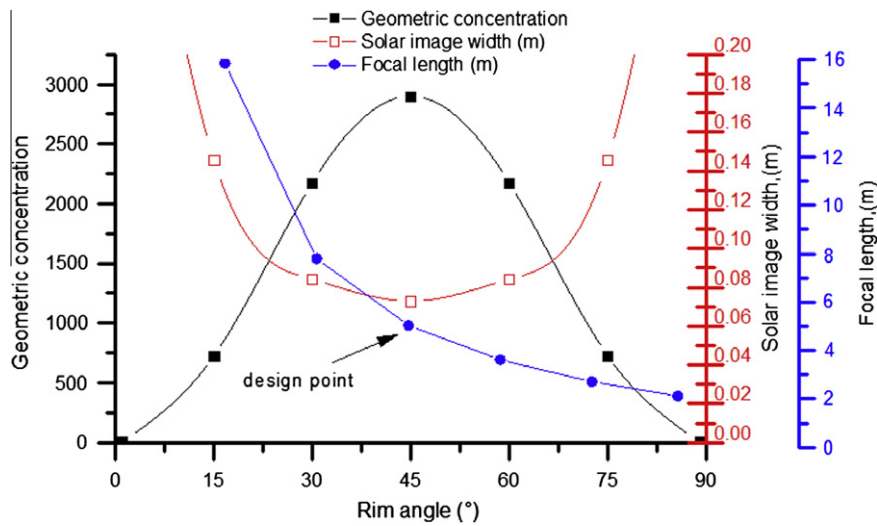


Fig. 6. Experimental determination of the focal length.

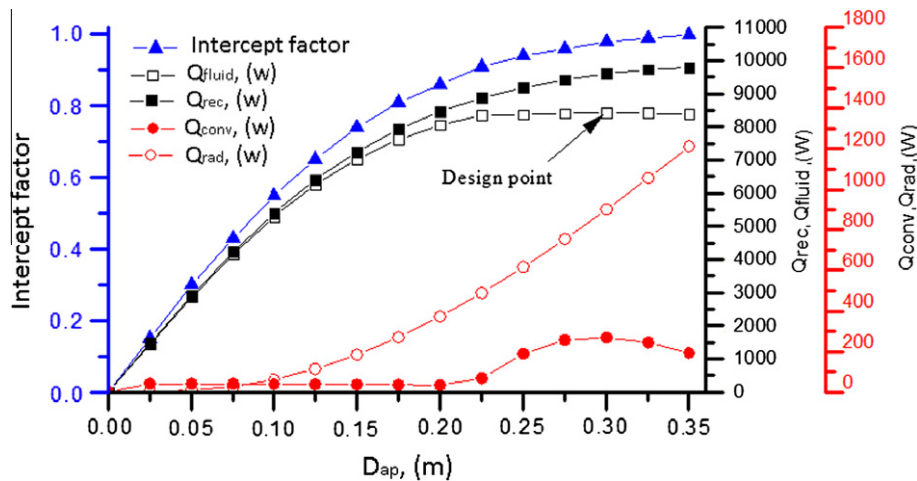


Fig. 7. Experimental determination of the receiver aperture opening diameter.

losses, which are transferred to the inner walls of the cavity, to the exterior walls and finally to the atmosphere by convection.

The mathematical model of the receiver achieves an energy balance, as shown in Eq. (11).

$$Q_{cav} = Q_{rec} - Q_{rad,ref} - Q_{rad,emi} - Q_{conv,nat} - Q_{conv,for} - Q_{cond,conv} \quad (11)$$

The heat loss by the reflected radiation is given by Eq. (12).

$$Q_{rad,ref} = (1 - \alpha_{eff}) Q_{rec} \quad (12)$$

where $(1 - \alpha_{eff})$ is the effective reflectance, shown in Eq. (13), of the cavity given by Duffie and Beckman [16].

$$\alpha_{eff} = \left(\frac{\alpha_{cav}}{\alpha_{cav} + (1 - \alpha_{cav}) \left(\frac{A_{ap,cav}}{A_{cav}} \right)} \right) \quad (13)$$

Radiation losses are calculated using Eq. (14), where $A_{ap,rec}$ is a function of the cavity's aperture opening diameter.

$$Q_{rad,emi} = \varepsilon_{cav}^* \sigma^* A_{ap,rec} (T_{cav}^4 - T_{amb}^4) \quad (14)$$

Natural convection losses are given by Eq. (15).

$$Q_{conv,nat} = h_{int,nat} A_{int,cav} (T_{cav} - T_{amb}) \quad (15)$$

To calculate the losses by natural convection in the cavity surface with Eq. (16), the correlation proposed by Stine et al. [26], which has been validated in other studies by Nepveu [27] and Sendhil [28], was used.

$$Nu_{int,nat} = 0.088 Gr^{1/3} (T_{cav}/T_{amb})^{0.18} \cos(\theta)^{2.47} (d_{ap}/d_{cav})^m, \text{ where} \\ m = -0.982 (d_{ap}/d_{cav}) + 1.12 \quad (16)$$

Forced convection losses are given by Eq. (17).

$$Q_{conv,for} = h_{int,for} A_{int,cav} (T_{cav} - T_{amb}) \quad (17)$$

To calculate the forced convection inside the cavity Eq. (18), factors such as wind and the slope of the cavity are considered, and we use the model proposed by Ma [29].

$$h_{int,for} = f(\theta) V^{1.401}; \text{ where } f(\theta) \\ = 0.163 + 0.749 \sin(\theta) - 0.502 \sin(2\theta) + 0.327 \\ \times \sin(3\theta) \quad (18)$$

The conduction heat loss in the cavity is transferred through the walls of the cavity and is then transferred to the environment by convection, in accordance with Eq. (19).

$$Q_{cond,conv} = \frac{(T_{cav} - T_{amb})}{L / (k_{ins} A_{int,cav} + 1 / (h_{ext,cav} A_{ext,cav}))} \quad (19)$$

The total convection heat transfer coefficient involves the natural and forced convection [26], as shown by Eq. (20).

$$h_{ext,cav} = (h_{ext,nat}^3 + h_{ext,for}^3)^{1/3} \quad (20)$$

To calculate the convection coefficient on the outer surface of the cavity when oriented vertically, the Nusselt number is given by Eq. (21) and Eq. (22) with respect to the correlation developed by Churchill and Chu and with reference to Incropera and DeWitt [30].

$$Nu_{ext,nat} = 0.27 Ra^{1/4}; (10^3 \leq Ra \leq 10^{10}; 60^\circ \leq \theta \leq 90^\circ) \quad (21)$$

$$Nu_{ext,for} = 0.664 Re^{1/2} Pr^{1/3} \quad (22)$$

Finally, the heat remaining in the cavity, Q_{cav} , is transferred to the fluid useful energy (see Fig. 4) through the walls of the absorber tubes, as shown by Eq. (23).

$$Q_{con,tub} = \frac{(T_{cav} - T_{int,tub})}{\log(d_{ext}/d_{in}) / (2\pi k_{tub} L_{abs} N_{tub,abs})} \quad (23)$$

Table 3

Parameters for comparison of a parabolic dish with respect to STSC.

Parameters	Parabolic dish	STSC
Diameter of the reflector	4 m	3.92 m
Rim angle	45°	45°
Focal length	2.6 m	5.04 m
Receiver diameter	0.36 m	0.36 m
Receiver aperture opening diameter	0.15 m	0.3 m
Cavity length	0.125 m	0.057 m
Inclination of the cavity	40° prom	80.7° fixed
Total error of the solar collector	8 mrad	12 mrad
Intercept factor	0.99	0.98
Radiation intercepted	10301 W	9597 W
Heat in the fluid	8423 W	8423 W
Receiver efficiency	81%	88%

3. Results and discussion

3.1. Model resolution

The mathematical model is solved using a convergent approximation algorithm in which there is a proposed initial temperature of the cavity and the temperature will increase gradually to reach an energy balance in the cavity receiver. Initially the geometric dimensions and properties of the concentration system and environmental conditions are introduced. Then, the intercept factor of the concentrator, the energy delivered to the receiver and the energy transferred to the atmosphere by convection, radiation and conduction are calculated to finally obtain the amount of energy transferred to the fluid. The algorithm is implemented in the program Engineering Equation Solver (EES).

3.2. Model validation

The validation process of the optical-geometric model is performed by comparing the experimental results for the WGA500 concentrator [31] and the results of the simulator of the geometric-optical model. To do this, the optical and geometric features of the WGA500 concentrator (focal length 4.5 m, diameter 3.8 m parabola, reflector) are introduced in the simulator. This type of concentrator is most appropriate for validating low-power solar concentrators, according to a study by Fraser [15]. Fig. 5 shows that the proposed model is modified by adjusting the exponential of the polynomial I (a) of Eq. (8) from a value of 1.5–1.15, resulting in a better approximation of the curve of the experimental results, coinciding with maximum variations of 10%, while the proposed model without modification presents variations greater than 10%. These results also show that for intercept factors greater than 0.9, the proposed modified model coincides with maximum variations of 1%. This result is adequate because the intercept factor is used precisely in this range as in this range, reductions in the heat losses caused by the excessive size of the opening of the cavity receiver are achieved.

The validation process of the thermal model is performed by comparing the experimental results reported for the 10 kW cavity receiver CNRS-PROMES system and the results obtained from simulation of the thermal model for this purpose. The thermal and geometric features of the 10 kW receiver CNRS-PROMES system (focal length 4.5 m, diameter 3.8 m parabola, reflector) [32] are introduced in the simulator. According to [15], this type of receptor is most appropriate for validating low-power receivers. Table 1 shows that the results of the proposed model are in the range established with respect to the experimental values. It is clear that the losses are not compared by conduction because this system considers the losses by conduction within the losses due to convec-

tion heat losses that are subsequently transferred by conduction to the environment by convection.

3.3. Numerical results

The scan sequence of parameters is performed to search for design parameters of the reflector using the mathematical model and the design parameters that correspond to the characteristics of the Stirling engine according to previous work [33] and the operating parameters for the weather conditions of Mexicali, Baja California, Mexico, which are shown in Table 2.

The model resolution consists of proposing a radius, for the reflector aperture area, and then this radius value is adjusted to obtain the quantity and quality of energy required by the 3 kWe Stirling engine, which corresponds to a thermal energy demand of 8423 W with a fluid temperature of 957 K, according to the study by Beltran et al. [33]. A radius of 1.96 m was found, which meets the quality and quantity of heat required by the Stirling engine. For this radius, the y-intercept value is 2.21 m, and the reflector diameter is 8.34 m for the parabola when considering a spacing of 0.25 m, as shown by Eq. (24).

$$d_{ref} = 2(2r + S) \quad (24)$$

Simultaneously, the model performs a scan to find the focal length and rim angle that provide the highest geometric concentration Eq. (25). Finding that the highest concentration corresponds to an rim angle of 45°.

$$C_{geo} = \frac{4\pi r^2}{\pi W_n^2} \quad (25)$$

This rim angle corresponds to a focal length of the parabola of 5.04 m because the rim angle of 45° provides for a smaller solar image in the focal plane, and then, the highest geometric concentration is obtained (Fig. 6).

Continuing the sequence design, a calculation is performed to determine the location of the absorber by calculating the length of the cavity Eq. (26) and the spacing of the absorber (S_p) with respect to the focal point Eq. (27), as observed in Fig. 2. The radiation is distributed best on the absorber surface with a cavity length of 5.72 cm and a spacing of 20.86 cm to comply with this condition, because the projection cone image is taken after the focal point.

$$L_{cav} = \frac{d_{cav} - d_{ap,rec} - Esp}{\tan(\psi_{sup})} \quad (26)$$

$$S_p = \frac{d_{ap,rec} - L_{cav} \tan(\psi_{inf})}{2} \quad (27)$$

The receiver aperture diameter is designed to transfer the most heat to the fluid, and this condition is obtained with an aperture diameter of 0.3 m and an intercept factor of 0.98. This finding is determined because, at this point, the aperture diameter increases, which not only increases the intercept factor but also further increases the loss due to radiation and convection losses caused by the excessive size of the aperture diameter (Fig. 7).

Table 3 shows the parameters used to size the STSC and the parabolic dish concentrator that satisfies the same thermal demand reported by Beltran and Velázquez [32]. To make a comparison between an STSC and a parabolic dish, the STSC operates with less precision but has a better thermal efficiency ($\eta = Q_{rec}/Q_{fluid}$), which is increased by 7%. This result is due to the inclination of the receiver angle and due to a decrease in the losses caused by convection; however, the focal length increases because of the relationship to the diameter of the parabola, and this relationship increases the area of the solar image at the focal point.

4. Conclusions

In this work, an STSC is designed to be coupled to a 3 kWe Stirling engine in a fixed position by introducing a new mathematical model that includes geometric, optical and thermal aspects. The model and methodology can be extrapolated to other applications by adjusting the design parameters and operation conditions.

The results allow for the determination of the focal length, which demonstrates that the largest concentration using the STSC with a rim angle of 45° is similar to the concentration achieved with a parabolic dish, but the receptor improves the efficiency by 7% compared with parabolic dishes that operate with a lower error of concentration. This result is due mainly to the case study that presents a receiver incline that favours the reduction convective heat loss. However, the relationship (f/d) results in a greater focal length for the STSC than the focal length of a parabolic dish. Consequently, the STSC is appropriate for regions that allow a reduction of heat losses and thermal energy requirements because the absorber area and focal length is adjusted to the focal image.

Finally, the new geometric model for estimating the intercept factor is versatile because it can be applied to both parabolic dish concentrators and STSCs by adjusting the correct limits of integration.

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